Mathematica 11.3 Integration Test Results

Test results for the 70 problems in "5.3.5 u (a+b arctan(c+d x))^p.m"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b\,\text{ArcTan}[c + d\,x]}{c\,e + d\,e\,x}\,dx$$

Optimal (type 4, 63 leaves, 5 steps):

$$\frac{a\,\text{Log}[c + d\,x]}{d\,e} + \frac{i\,b\,\text{PolyLog}[2, -i\,(c + d\,x)]}{2\,d\,e} - \frac{i\,b\,\text{PolyLog}[2, i\,(c + d\,x)]}{2\,d\,e}$$

Result (type 4, 189 leaves):

$$-\frac{1}{8\,d\,e}\Big(i\,b\,\pi^2 - 4\,i\,b\,\pi\,\text{ArcTan}[c + d\,x] + 8\,i\,b\,\text{ArcTan}[c + d\,x]^2 + b\,\pi\,\text{Log}[16] - 4\,b\,\pi\,\text{Log}\big[1 + e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big] + 8\,b\,\text{ArcTan}[c + d\,x]\,\text{Log}\big[1 + e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big] - 8\,b\,\text{ArcTan}[c + d\,x]\,\text{Log}\big[1 - e^{2\,i\,\text{ArcTan}[c+d\,x]}\big] - 8\,a\,\text{Log}[c + d\,x] - 2\,b\,\pi\,\text{Log}\big[1 + c^2 + 2\,c\,d\,x + d^2\,x^2\big] + 4\,i\,b\,\text{PolyLog}\big[2, -e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big] + 4\,i\,b\,\text{PolyLog}\big[2, e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]\Big)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b\,\text{ArcTan}[c + d\,x])^2}{c\,e + d\,e\,x}\,dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$\frac{2\,(a + b\,\text{ArcTan}[c + d\,x])^2\,\text{ArcTanh}\Big[1 - \frac{2}{1+i\,(c+d\,x)}\Big]}{d\,e} - \frac{i\,b\,(a + b\,\text{ArcTan}[c + d\,x])\,\text{PolyLog}\Big[2, 1 - \frac{2}{1+i\,(c+d\,x)}\Big]}{d\,e} + \frac{i\,b\,(a + b\,\text{ArcTan}[c + d\,x])\,\text{PolyLog}\Big[2, -1 + \frac{2}{1+i\,(c+d\,x)}\Big]}{d\,e} - \frac{b^2\,\text{PolyLog}\Big[3, 1 - \frac{2}{1+i\,(c+d\,x)}\Big]}{2\,d\,e} + \frac{b^2\,\text{PolyLog}\Big[3, -1 + \frac{2}{1+i\,(c+d\,x)}\Big]}{2\,d\,e}$$

Result (type 4, 381 leaves):

$$\frac{1}{24\,d\,e}\Big(-6\,i\,a\,b\,\pi^2 - i\,b^2\,\pi^3 + 24\,i\,a\,b\,\pi\,\text{ArcTan}[c+d\,x] - 48\,i\,a\,b\,\text{ArcTan}[c+d\,x]^2 + 16\,i\,b^2\,\text{ArcTan}[c+d\,x]^3 - a\,b\,\pi\,\text{Log}[16\,777\,216] + 24\,b^2\,\text{ArcTan}[c+d\,x]^2\,\text{Log}\left[1-e^{-2\,i\,\text{ArcTan}[c+d\,x]}\right] + 24\,a\,b\,\pi\,\text{Log}\left[1+e^{-2\,i\,\text{ArcTan}[c+d\,x]}\right] - 48\,a\,b\,\text{ArcTan}[c+d\,x]\,\text{Log}\left[1+e^{-2\,i\,\text{ArcTan}[c+d\,x]}\right] + 48\,a\,b\,\text{ArcTan}[c+d\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcTan}[c+d\,x]}\right] - 24\,b^2\,\text{ArcTan}[c+d\,x]^2\,\text{Log}\left[1+e^{2\,i\,\text{ArcTan}[c+d\,x]}\right] + 24\,a^2\,\text{Log}[c+d\,x] + 12\,a\,b\,\pi\,\text{Log}\left[1+c^2+2\,c\,d\,x+d^2\,x^2\right] - 24\,i\,a\,b\,\text{PolyLog}\left[2,-e^{-2\,i\,\text{ArcTan}[c+d\,x]}\right] + 24\,i\,b^2\,\text{ArcTan}[c+d\,x]\,\text{PolyLog}\left[2,e^{-2\,i\,\text{ArcTan}[c+d\,x]}\right] + 24\,i\,b^2\,\text{ArcTan}[c+d\,x]\,\text{PolyLog}\left[2,-e^{2\,i\,\text{ArcTan}[c+d\,x]}\right] - 24\,i\,a\,b\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcTan}[c+d\,x]}\right] + 12\,b^2\,\text{PolyLog}\left[3,e^{-2\,i\,\text{ArcTan}[c+d\,x]}\right] - 12\,b^2\,\text{PolyLog}\left[3,-e^{2\,i\,\text{ArcTan}[c+d\,x]}\right]\Big)$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\text{ArcTan}[c+d\,x])^3}{c\,e+d\,e\,x}\,dx$$

Optimal (type 4, 279 leaves, 10 steps):

$$\frac{2\,(a+b\,\text{ArcTan}[c+d\,x])^3\,\text{ArcTanh}\left[1-\frac{2}{1+i\,(c+d\,x)}\right]}{d\,e} - \frac{3\,i\,b\,(a+b\,\text{ArcTan}[c+d\,x])^2\,\text{PolyLog}\left[2,1-\frac{2}{1+i\,(c+d\,x)}\right]}{2\,d\,e} + \frac{3\,i\,b\,(a+b\,\text{ArcTan}[c+d\,x])^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+i\,(c+d\,x)}\right]}{2\,d\,e} - \frac{3\,b^2\,(a+b\,\text{ArcTan}[c+d\,x])\,\text{PolyLog}\left[3,1-\frac{2}{1+i\,(c+d\,x)}\right]}{2\,d\,e} + \frac{3\,b^2\,(a+b\,\text{ArcTan}[c+d\,x])\,\text{PolyLog}\left[3,-1+\frac{2}{1+i\,(c+d\,x)}\right]}{2\,d\,e} + \frac{3\,i\,b^3\,\text{PolyLog}\left[4,1-\frac{2}{1+i\,(c+d\,x)}\right]}{4\,d\,e} - \frac{3\,i\,b^3\,\text{PolyLog}\left[4,-1+\frac{2}{1+i\,(c+d\,x)}\right]}{4\,d\,e}$$

Result (type 4, 562 leaves):

$$\frac{1}{64\,d\,e}\Big(64\,a^3\,\text{Log}[c+d\,x]-24\,i\,a^2\,b\,\big(\pi^2-4\,\pi\,\text{ArcTan}[c+d\,x]+8\,\text{ArcTan}[c+d\,x]^2-i\,\pi\,\text{Log}[16]+4\,i\,\pi\,\text{Log}\big[1+e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]-8\,i\,\text{ArcTan}[c+d\,x]\,\text{Log}\big[1+e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]+8\,i\,\text{ArcTan}[c+d\,x]\,\text{Log}\big[1-e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]+2\,i\,\pi\,\text{Log}\big[1+c^2+2\,c\,d\,x+d^2\,x^2\big]+4\,\text{PolyLog}\big[2,-e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]+4\,\text{PolyLog}\big[2,e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]\big)+8\,a\,b^2\,\big(-i\,\pi^3+16\,i\,\text{ArcTan}[c+d\,x]^3+24\,\text{ArcTan}[c+d\,x]^2\,\text{Log}\big[1-e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]-24\,\text{ArcTan}[c+d\,x]^2\,\text{Log}\big[1+e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]+24\,i\,\text{ArcTan}[c+d\,x]\,\text{PolyLog}\big[2,e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]+24\,i\,\text{ArcTan}[c+d\,x]\,\text{PolyLog}\big[2,-e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]+12\,\text{PolyLog}\big[3,e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]-12\,\text{PolyLog}\big[3,-e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]\big)-i\,b^3\,\big(\pi^4-32\,\text{ArcTan}[c+d\,x]^4+64\,i\,\text{ArcTan}[c+d\,x]^3\,\text{Log}\big[1-e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]-64\,i\,\text{ArcTan}[c+d\,x]^3\,\text{Log}\big[1+e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]-96\,\text{ArcTan}[c+d\,x]^2\,\text{PolyLog}\big[2,e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]-96\,\text{ArcTan}[c+d\,x]^2\,\text{PolyLog}\big[2,-e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]+96\,i\,\text{ArcTan}[c+d\,x]\,\text{PolyLog}\big[3,e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]-96\,i\,\text{ArcTan}[c+d\,x]\,\text{PolyLog}\big[3,-e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]+48\,\text{PolyLog}\big[4,e^{-2\,i\,\text{ArcTan}[c+d\,x]}\big]+48\,\text{PolyLog}\big[4,-e^{2\,i\,\text{ArcTan}[c+d\,x]}\big]\big)\Big)$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTan}[1+x]}{2+2\,x}\,dx$$

Optimal (type 4, 31 leaves, 5 steps):

$$\frac{1}{4}\,i\,\text{PolyLog}[2,-i\,(1+x)]-\frac{1}{4}\,i\,\text{PolyLog}[2,i\,(1+x)]$$

Result (type 4, 138 leaves):

$$-\frac{1}{16}\,i\,\Big(\pi^2-4\,\pi\,\text{ArcTan}[1+x]+8\,\text{ArcTan}[1+x]^2-i\,\pi\,\text{Log}[16]+4\,i\,\pi\,\text{Log}\big[1+e^{-2\,i\,\text{ArcTan}[1+x]}\big]-8\,i\,\text{ArcTan}[1+x]\,\text{Log}\big[1+e^{-2\,i\,\text{ArcTan}[1+x]}\big]+8\,i\,\text{ArcTan}[1+x]\,\text{Log}\big[1-e^{2\,i\,\text{ArcTan}[1+x]}\big]+2\,i\,\pi\,\text{Log}\big[2+2\,x+x^2\big]+4\,\text{PolyLog}\big[2,-e^{-2\,i\,\text{ArcTan}[1+x]}\big]+4\,\text{PolyLog}\big[2,e^{2\,i\,\text{ArcTan}[1+x]}\big]\Big)$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTan}[a+b\,x]}{\frac{a\,d}{b}+d\,x}\,dx$$

Optimal (type 4, 41 leaves, 5 steps):

$$\frac{i\,\text{PolyLog}[2,-i\,(a+b\,x)]}{2\,d}-\frac{i\,\text{PolyLog}[2,i\,(a+b\,x)]}{2\,d}$$

Result (type 4, 168 leaves):

$$-\frac{1}{8\,d}\,i\,\Big(\pi^2-4\,\pi\,\text{ArcTan}[a+b\,x]+8\,\text{ArcTan}[a+b\,x]^2-i\,\pi\,\text{Log}[16]+4\,i\,\pi\,\text{Log}\big[1+e^{-2\,i\,\text{ArcTan}[a+b\,x]}\big]-8\,i\,\text{ArcTan}[a+b\,x]\,\text{Log}\big[1+e^{-2\,i\,\text{ArcTan}[a+b\,x]}\big]+8\,i\,\text{ArcTan}[a+b\,x]\,\text{Log}\big[1-e^{2\,i\,\text{ArcTan}[a+b\,x]}\big]+2\,i\,\pi\,\text{Log}\big[1+a^2+2\,a\,b\,x+b^2\,x^2\big]+4\,\text{PolyLog}\big[2,-e^{-2\,i\,\text{ArcTan}[a+b\,x]}\big]+4\,\text{PolyLog}\big[2,e^{2\,i\,\text{ArcTan}[a+b\,x]}\big]\Big)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int (e+f x)^2 (a+b \text{ArcTan}[c+d x])^2 \, dx$$

Optimal (type 4, 382 leaves, 16 steps):

$$\frac{b^2 f^2 x}{3 d^2}-\frac{2 a b f (d e-c f) x}{d^2}-\frac{b^2 f^2 \text{ArcTan}[c+d x]}{3 d^3}-\frac{2 b^2 f (d e-c f) (c+d x) \text{ArcTan}[c+d x]}{d^3}-\frac{b f^2 (c+d x)^2 (a+b \text{ArcTan}[c+d x])}{3 d^3}+\frac{i \left(3 d^2 e^2-6 c d e f-\left(1-3 c^2\right) f^2\right) (a+b \text{ArcTan}[c+d x])^2}{3 d^3}-\frac{(d e-c f) \left(d^2 e^2-2 c d e f-\left(3-c^2\right) f^2\right) (a+b \text{ArcTan}[c+d x])^2}{3 d^3 f}+\frac{(e+f x)^3 (a+b \text{ArcTan}[c+d x])^2}{3 f}+\frac{1}{3 d^3} 2 b \left(3 d^2 e^2-6 c d e f-\left(1-3 c^2\right) f^2\right) (a+b \text{ArcTan}[c+d x]) \text{Log}\left[\frac{2}{1+i (c+d x)}\right]+\frac{b^2 f (d e-c f) \text{Log}\left[1+(c+d x)^2\right]}{d^3}+\frac{i b^2 \left(3 d^2 e^2-6 c d e f-\left(1-3 c^2\right) f^2\right) \text{PolyLog}\left[2,1-\frac{2}{1+i (c+d x)}\right]}{3 d^3}$$

Result (type 4, 801 leaves):

$$a^2 e^2 x + a^2 e f x^2 + \frac{1}{3} a^2 f^2 x^3 + \frac{1}{3 d^3} a b \left(-d f x \left(6 d e - 4 c f + d f x\right) + 2 \left(3 d e f - 3 c^2 d e f + c^3 f^2 + 3 c \left(d^2 e^2 - f^2\right) + d^3 x \left(3 e^2 + 3 e f x + f^2 x^2\right)\right) \text{ArcTan}[c + d x] + \left(-3 d^2 e^2 + 6 c d e f + \left(1 - 3 c^2\right) f^2\right) \text{Log}\left[1 + (c + d x)^2\right]\right) + \frac{1}{d} b^2 e^2 \left(\text{ArcTan}[c + d x] \left(\left(-i + c + d x\right) \text{ArcTan}[c + d x] + 2 \text{Log}\left[1 + e^{2 i \text{ArcTan}[c+d x]}\right]\right) - i \text{PolyLog}\left[2, -e^{2 i \text{ArcTan}[c+d x]}\right]\right) + \frac{1}{d^2} b^2 e f \left(\left(1 + 2 i c - c^2 + d^2 x^2\right) \text{ArcTan}[c + d x]^2 - 2 \text{ArcTan}[c + d x] \left(c + d x + 2 c \text{Log}\left[1 + e^{2 i \text{ArcTan}[c+d x]}\right]\right) + \text{Log}\left[1 + (c + d x)^2\right] + 2 i c \text{PolyLog}\left[2, -e^{2 i \text{ArcTan}[c+d x]}\right]\right) +$$

$$\frac{1}{12 d^3} b^2 f^2 \left(1 + (c + d x)^2\right)^{3/2} \left(\frac{c + d x}{\sqrt{1 + (c + d x)^2}} + \frac{6 c (c + d x) \text{ArcTan}[c + d x]}{\sqrt{1 + (c + d x)^2}} + \frac{3 (c + d x) \text{ArcTan}[c + d x]^2}{\sqrt{1 + (c + d x)^2}} + \frac{3 c^2 (c + d x) \text{ArcTan}[c + d x]^2}{\sqrt{1 + (c + d x)^2}} + i \text{ArcTan}[c + d x]^2 \text{Cos}[3 \text{ArcTan}[c + d x]] - 3 i c^2 \text{ArcTan}[c + d x]^2 \text{Cos}[3 \text{ArcTan}[c + d x]] - 2 \text{ArcTan}[c + d x] \text{Cos}[3 \text{ArcTan}[c + d x]] \text{Log}\left[1 + e^{2 i \text{ArcTan}[c+d x]}\right] + 6 c^2 \text{ArcTan}[c + d x] \text{Cos}[3 \text{ArcTan}[c + d x]] \text{Log}\left[1 + e^{2 i \text{ArcTan}[c+d x]}\right] + 6 c \text{Cos}[3 \text{ArcTan}[c + d x]] \text{Log}\left[\frac{1}{\sqrt{1 + (c + d x)^2}}\right] + \frac{1}{\sqrt{1 + (c + d x)^2}} \left(\left(3 i - 12 c - 9 i c^2\right) \text{ArcTan}[c + d x]^2 + 2 \text{ArcTan}[c + d x] \left(-2 + \left(-3 + 9 c^2\right) \text{Log}\left[1 + e^{2 i \text{ArcTan}[c+d x]}\right]\right) + 18 c \text{Log}\left[\frac{1}{\sqrt{1 + (c + d x)^2}}\right]\right) - \frac{4 i \left(-1 + 3 c^2\right) \text{PolyLog}\left[2, -e^{2 i \text{ArcTan}[c+d x]}\right]}{\left(1 + (c + d x)^2\right)^{3/2}} + \text{Sin}[3 \text{ArcTan}[c + d x]] + 6 c \text{ArcTan}[c + d x] \text{Sin}[3 \text{ArcTan}[c + d x]] - \text{ArcTan}[c + d x]^2 \text{Sin}[3 \text{ArcTan}[c + d x]] + 3 c^2 \text{ArcTan}[c + d x]^2 \text{Sin}[3 \text{ArcTan}[c + d x]]\right)$$

Problem 34: Unable to integrate problem.

$$\int \frac{(a + b \text{ArcTan}[c + d x])^2}{e + f x} \, dx$$

Optimal (type 4, 261 leaves, 2 steps):

$$-\frac{\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2\,\text{Log}\left[\frac{2}{1-i\,(c+d\,x)}\right]}{f}+\frac{\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2\,\text{Log}\left[\frac{2\,d\,(e+f\,x)}{(d\,e+i\,f-c\,f)\,(1-i\,(c+d\,x))}\right]}{f}+$$
$$\frac{i\,b\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)\,\text{PolyLog}\left[2,1-\frac{2}{1-i\,(c+d\,x)}\right]}{f}-$$
$$\frac{i\,b\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)\,\text{PolyLog}\left[2,1-\frac{2\,d\,(e+f\,x)}{(d\,e+i\,f-c\,f)\,(1-i\,(c+d\,x))}\right]}{f}-$$
$$\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1-i\,(c+d\,x)}\right]}{2\,f}+\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2\,d\,(e+f\,x)}{(d\,e+i\,f-c\,f)\,(1-i\,(c+d\,x))}\right]}{2\,f}$$

Result (type 8, 22 leaves):

$$\int\frac{\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2}{e+f\,x}\,dx$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int(e+f\,x)^2\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^3\,dx$$

Optimal (type 4, 564 leaves, 21 steps):

$$\frac{a\,b^2\,f^2\,x}{d^2}+\frac{b^3\,f^2\,(c+d\,x)\,\text{ArcTan}[c+d\,x]}{d^3}-$$
$$\frac{b\,f^2\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2}{2\,d^3}-\frac{3\,i\,b\,f\,(d\,e-c\,f)\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2}{d^3}-$$
$$\frac{3\,b\,f\,(d\,e-c\,f)\,(c+d\,x)\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2}{d^3}-\frac{b\,f^2\,(c+d\,x)^2\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2}{2\,d^3}+$$
$$\frac{i\,\left(3\,d^2\,e^2-6\,c\,d\,e\,f-\left(1-3\,c^2\right)\,f^2\right)\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^3}{3\,d^3}-$$
$$\frac{(d\,e-c\,f)\,\left(d^2\,e^2-2\,c\,d\,e\,f-\left(3-c^2\right)\,f^2\right)\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^3}{3\,d^3\,f}+$$
$$\frac{(e+f\,x)^3\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^3}{3\,f}-\frac{6\,b^2\,f\,(d\,e-c\,f)\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)\,\text{Log}\left[\frac{2}{1+i\,(c+d\,x)}\right]}{d^3}+$$
$$\frac{1}{d^3}b\,\left(3\,d^2\,e^2-6\,c\,d\,e\,f-\left(1-3\,c^2\right)\,f^2\right)\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)^2\,\text{Log}\left[\frac{2}{1+i\,(c+d\,x)}\right]-$$
$$\frac{b^3\,f^2\,\text{Log}\left[1+(c+d\,x)^2\right]}{2\,d^3}-\frac{3\,i\,b^3\,f\,(d\,e-c\,f)\,\text{PolyLog}\left[2,1-\frac{2}{1+i\,(c+d\,x)}\right]}{d^3}+\frac{1}{d^3}$$
$$i\,b^2\,\left(3\,d^2\,e^2-6\,c\,d\,e\,f-\left(1-3\,c^2\right)\,f^2\right)\,\left(a+b\,\text{ArcTan}[c+d\,x]\right)\,\text{PolyLog}\left[2,1-\frac{2}{1+i\,(c+d\,x)}\right]+$$
$$\frac{b^3\,\left(3\,d^2\,e^2-6\,c\,d\,e\,f-\left(1-3\,c^2\right)\,f^2\right)\,\text{PolyLog}\left[3,1-\frac{2}{1+i\,(c+d\,x)}\right]}{2\,d^3}$$

Result (type 4, 1839 leaves):

$$\frac{a^2\left(a d^2 e^2-3 b d e f+2 b c f^2\right) x}{d^2}-\frac{a^2 f\left(-2 a d e+b f\right) x^2}{2 d}+\frac{1}{3} a^3 f^2 x^3+\frac{1}{d^3}$$
$$\left(3 a^2 b c d^2 e^2+3 a^2 b d e f-3 a^2 b c^2 d e f-3 a^2 b c f^2+a^2 b c^3 f^2\right) \text{ArcTan}[c+d x]+$$
$$a^2 b x\left(3 e^2+3 e f x+f^2 x^2\right) \text{ArcTan}[c+d x]+\frac{1}{2 d^3}$$
$$\left(-3 a^2 b d^2 e^2+6 a^2 b c d e f+a^2 b f^2-3 a^2 b c^2 f^2\right) \text{Log}\left[1+c^2+2 c d x+d^2 x^2\right]+$$
$$6 a b^2 e f\left(-\frac{(c+d x) \text{ArcTan}[c+d x]}{d^2}-\frac{c\,(c+d x) \text{ArcTan}[c+d x]^2}{d^2}+\right.$$
$$\frac{\left(1+(c+d x)^2\right) \text{ArcTan}[c+d x]^2}{2 d^2}-\frac{\text{Log}\left[\frac{1}{\sqrt{1+(c+d x)^2}}\right]}{d^2}+\frac{1}{d^2} 2 c\left(\frac{1}{2} i\, \text{ArcTan}[c+d x]^2-\right.$$
$$\left.\left.\text{ArcTan}[c+d x] \text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]+\frac{1}{2} i\, \text{PolyLog}\left[2,-e^{2 i \text{ArcTan}[c+d x]}\right]\right)\right)+\frac{1}{d} 3 a b^2 e^2$$
$$\left(\text{ArcTan}[c+d x]\left(-i\, \text{ArcTan}[c+d x]+(c+d x) \text{ArcTan}[c+d x]+2 \text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]\right)-\right.$$
$$\left.i\, \text{PolyLog}\left[2,-e^{2 i \text{ArcTan}[c+d x]}\right]\right)+\frac{1}{d}$$
$$b^3 e^2\left(\text{ArcTan}[c+d x]^2\left(-i\, \text{ArcTan}[c+d x]+(c+d x) \text{ArcTan}[c+d x]+3 \text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]\right)-\right.$$
$$\left.3 i\, \text{ArcTan}[c+d x] \text{PolyLog}\left[2,-e^{2 i \text{ArcTan}[c+d x]}\right]+\frac{3}{2} \text{PolyLog}\left[3,-e^{2 i \text{ArcTan}[c+d x]}\right]\right)+$$
$$\frac{1}{d^2} b^3 e f\left(\text{ArcTan}[c+d x]\left(3 i\, \text{ArcTan}[c+d x]+2 i c\, \text{ArcTan}[c+d x]^2+\right.\right.$$
$$\left(1+(c+d x)^2\right) \text{ArcTan}[c+d x]^2-(c+d x) \text{ArcTan}[c+d x]\left(3+2 c\, \text{ArcTan}[c+d x]\right)-$$
$$\left.6 \text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]-6 c\, \text{ArcTan}[c+d x] \text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]\right)+$$
$$\left.3 i\left(1+2 c\, \text{ArcTan}[c+d x]\right) \text{PolyLog}\left[2,-e^{2 i \text{ArcTan}[c+d x]}\right]-3 c\, \text{PolyLog}\left[3,-e^{2 i \text{ArcTan}[c+d x]}\right]\right)+$$
$$\frac{1}{4 d^3} a b^2 f^2\left(1+(c+d x)^2\right)^{3/2}\left(\frac{c+d x}{\sqrt{1+(c+d x)^2}}+\frac{6 c\,(c+d x) \text{ArcTan}[c+d x]}{\sqrt{1+(c+d x)^2}}+\right.$$
$$\frac{3\,(c+d x) \text{ArcTan}[c+d x]^2}{\sqrt{1+(c+d x)^2}}+\frac{3 c^2\,(c+d x) \text{ArcTan}[c+d x]^2}{\sqrt{1+(c+d x)^2}}+$$
$$i\, \text{ArcTan}[c+d x]^2 \text{Cos}[3 \text{ArcTan}[c+d x]]-3 i c^2 \text{ArcTan}[c+d x]^2 \text{Cos}[3 \text{ArcTan}[c+d x]]-$$
$$2 \text{ArcTan}[c+d x] \text{Cos}[3 \text{ArcTan}[c+d x]] \text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]+$$
$$6 c^2 \text{ArcTan}[c+d x] \text{Cos}[3 \text{ArcTan}[c+d x]] \text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]+$$
$$6 c\, \text{Cos}[3 \text{ArcTan}[c+d x]] \text{Log}\left[\frac{1}{\sqrt{1+(c+d x)^2}}\right]+\frac{1}{\sqrt{1+(c+d x)^2}}$$
$$\left(\text{ArcTan}[c+d x]\left(-4+\left(3 i-12 c-9 i c^2\right) \text{ArcTan}[c+d x]\right)+\right.$$

$$6\left(-1+3c^2\right)\text{ArcTan}[c+d x]\,\text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]+18 c\,\text{Log}\left[\frac{1}{\sqrt{1+(c+d x)^2}}\right]\Bigg)-$$

$$\frac{4 i\left(-1+3c^2\right)\text{PolyLog}\left[2,-e^{2 i \text{ArcTan}[c+d x]}\right]}{\left(1+(c+d x)^2\right)^{3/2}}+\text{Sin}[3\,\text{ArcTan}[c+d x]]+$$

$$6 c\,\text{ArcTan}[c+d x]\,\text{Sin}[3\,\text{ArcTan}[c+d x]]-\text{ArcTan}[c+d x]^2\,\text{Sin}[3\,\text{ArcTan}[c+d x]]+$$

$$3 c^2\,\text{ArcTan}[c+d x]^2\,\text{Sin}[3\,\text{ArcTan}[c+d x]]\Bigg)+$$

$$\frac{1}{d^3} b^3 f^2\Bigg(-i\left(3c-\text{ArcTan}[c+d x]+3c^2\,\text{ArcTan}[c+d x]\right)\text{PolyLog}\left[2,-e^{2 i \text{ArcTan}[c+d x]}\right]+$$

$$\frac{1}{12}\left(1+(c+d x)^2\right)^{3/2}$$

$$\Bigg(\frac{3(c+d x)\,\text{ArcTan}[c+d x]}{\sqrt{1+(c+d x)^2}}+\frac{9c(c+d x)\,\text{ArcTan}[c+d x]^2}{\sqrt{1+(c+d x)^2}}+\frac{3(c+d x)\,\text{ArcTan}[c+d x]^3}{\sqrt{1+(c+d x)^2}}+$$

$$\frac{3c^2(c+d x)\,\text{ArcTan}[c+d x]^3}{\sqrt{1+(c+d x)^2}}-9 i c\,\text{ArcTan}[c+d x]^2\,\text{Cos}[3\,\text{ArcTan}[c+d x]]+$$

$$i\,\text{ArcTan}[c+d x]^3\,\text{Cos}[3\,\text{ArcTan}[c+d x]]-3 i c^2\,\text{ArcTan}[c+d x]^3\,\text{Cos}[3\,\text{ArcTan}[c+d x]]+$$

$$18 c\,\text{ArcTan}[c+d x]\,\text{Cos}[3\,\text{ArcTan}[c+d x]]\,\text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]-3\,\text{ArcTan}[c+d x]^2\,\text{Cos}[3\,\text{ArcTan}[c+d x]]\,\text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]+9c^2\,\text{ArcTan}[c+d x]^2\,\text{Cos}[3\,\text{ArcTan}[c+d x]]$$

$$\text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]+3\,\text{Cos}[3\,\text{ArcTan}[c+d x]]\,\text{Log}\left[\frac{1}{\sqrt{1+(c+d x)^2}}\right]+$$

$$\Bigg(3\Bigg(\text{ArcTan}[c+d x]^2\left(-2-9 i c+i\,\text{ArcTan}[c+d x]-4c\,\text{ArcTan}[c+d x]-3 i c^2\,\text{ArcTan}[c+d x]\right)+3\,\text{ArcTan}[c+d x]\left(6c-\text{ArcTan}[c+d x]+3c^2\,\text{ArcTan}[c+d x]\right)$$

$$\text{Log}\left[1+e^{2 i \text{ArcTan}[c+d x]}\right]+3\,\text{Log}\left[\frac{1}{\sqrt{1+(c+d x)^2}}\right]\Bigg)\Bigg)\Big/\left(\sqrt{1+(c+d x)^2}\right)+$$

$$\frac{6\left(-1+3c^2\right)\text{PolyLog}\left[3,-e^{2 i \text{ArcTan}[c+d x]}\right]}{\left(1+(c+d x)^2\right)^{3/2}}+3\,\text{ArcTan}[c+d x]\,\text{Sin}[3\,\text{ArcTan}[c+d x]]+$$

$$9c\,\text{ArcTan}[c+d x]^2\,\text{Sin}[3\,\text{ArcTan}[c+d x]]-\text{ArcTan}[c+d x]^3\,\text{Sin}[3\,\text{ArcTan}[c+d x]]+$$

$$3c^2\,\text{ArcTan}[c+d x]^3\,\text{Sin}[3\,\text{ArcTan}[c+d x]]\Bigg)\Bigg)$$

Problem 39: Unable to integrate problem.

$$\int \frac{(a + b\,\text{ArcTan}[c + d\,x])^3}{e + f\,x}\,dx$$

Optimal (type 4, 372 leaves, 2 steps):

$$-\frac{(a + b\,\text{ArcTan}[c + d\,x])^3\,\text{Log}\left[\frac{2}{1-i\,(c+d\,x)}\right]}{f} + \frac{(a + b\,\text{ArcTan}[c + d\,x])^3\,\text{Log}\left[\frac{2\,d\,(e+f\,x)}{(d\,e+i\,f-c\,f)\,(1-i\,(c+d\,x))}\right]}{f} + \frac{3\,i\,b\,(a + b\,\text{ArcTan}[c + d\,x])^2\,\text{PolyLog}\left[2, 1 - \frac{2}{1-i\,(c+d\,x)}\right]}{2\,f} - \frac{3\,i\,b\,(a + b\,\text{ArcTan}[c + d\,x])^2\,\text{PolyLog}\left[2, 1 - \frac{2\,d\,(e+f\,x)}{(d\,e+i\,f-c\,f)\,(1-i\,(c+d\,x))}\right]}{2\,f} - \frac{3\,b^2\,(a + b\,\text{ArcTan}[c + d\,x])\,\text{PolyLog}\left[3, 1 - \frac{2}{1-i\,(c+d\,x)}\right]}{2\,f} + \frac{3\,b^2\,(a + b\,\text{ArcTan}[c + d\,x])\,\text{PolyLog}\left[3, 1 - \frac{2\,d\,(e+f\,x)}{(d\,e+i\,f-c\,f)\,(1-i\,(c+d\,x))}\right]}{2\,f} - \frac{3\,i\,b^3\,\text{PolyLog}\left[4, 1 - \frac{2}{1-i\,(c+d\,x)}\right]}{4\,f} + \frac{3\,i\,b^3\,\text{PolyLog}\left[4, 1 - \frac{2\,d\,(e+f\,x)}{(d\,e+i\,f-c\,f)\,(1-i\,(c+d\,x))}\right]}{4\,f}$$

Result (type 8, 22 leaves):

$$\int \frac{(a + b\,\text{ArcTan}[c + d\,x])^3}{e + f\,x}\,dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{(a + b\,\text{ArcTan}[c + d\,x])^3}{(e + f\,x)^2}\,dx$$

Optimal (type 4, 1233 leaves, 35 steps):

$$\frac{3 a^2 b d (d e - c f) \text{ArcTan}[c + d x]}{f (f^2 + (d e - c f)^2)} + \frac{3 i a b^2 d \text{ArcTan}[c + d x]^2}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} +$$

$$\frac{3 a b^2 d (d e - c f) \text{ArcTan}[c + d x]^2}{f (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} + \frac{i b^3 d \text{ArcTan}[c + d x]^3}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{b^3 d (d e - c f) \text{ArcTan}[c + d x]^3}{f (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} -$$

$$\frac{(a + b \text{ArcTan}[c + d x])^3}{f (e + f x)} + \frac{3 a^2 b d \text{Log}[e + f x]}{f^2 + (d e - c f)^2} - \frac{6 a b^2 d \text{ArcTan}[c + d x] \text{Log}\left[\frac{2}{1-i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} -$$

$$\frac{3 b^3 d \text{ArcTan}[c + d x]^2 \text{Log}\left[\frac{2}{1-i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{6 a b^2 d \text{ArcTan}[c + d x] \text{Log}\left[\frac{2 d (e+f x)}{(d e+i f-c f) (1-i (c+d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} +$$

$$\frac{3 b^3 d \text{ArcTan}[c + d x]^2 \text{Log}\left[\frac{2 d (e+f x)}{(d e+i f-c f) (1-i (c+d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{6 a b^2 d \text{ArcTan}[c + d x] \text{Log}\left[\frac{2}{1+i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} +$$

$$\frac{3 b^3 d \text{ArcTan}[c + d x]^2 \text{Log}\left[\frac{2}{1+i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \frac{3 a^2 b d \text{Log}\left[1 + (c + d x)^2\right]}{2 (f^2 + (d e - c f)^2)} +$$

$$\frac{3 i a b^2 d \text{PolyLog}\left[2, 1 - \frac{2}{1-i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{3 i b^3 d \text{ArcTan}[c + d x] \text{PolyLog}\left[2, 1 - \frac{2}{1-i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} -$$

$$\frac{3 i a b^2 d \text{PolyLog}\left[2, 1 - \frac{2 d (e+f x)}{(d e+i f-c f) (1-i (c+d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} -$$

$$\frac{3 i b^3 d \text{ArcTan}[c + d x] \text{PolyLog}\left[2, 1 - \frac{2 d (e+f x)}{(d e+i f-c f) (1-i (c+d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{3 i a b^2 d \text{PolyLog}\left[2, 1 - \frac{2}{1+i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} +$$

$$\frac{3 i b^3 d \text{ArcTan}[c + d x] \text{PolyLog}\left[2, 1 - \frac{2}{1+i (c+d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \frac{3 b^3 d \text{PolyLog}\left[3, 1 - \frac{2}{1-i (c+d x)}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} +$$

$$\frac{3 b^3 d \text{PolyLog}\left[3, 1 - \frac{2 d (e+f x)}{(d e+i f-c f) (1-i (c+d x))}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} + \frac{3 b^3 d \text{PolyLog}\left[3, 1 - \frac{2}{1+i (c+d x)}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)}$$

Result (type 8, 22 leaves):

$$\int \frac{(a + b \text{ArcTan}[c + d x])^3}{(e + f x)^2} \, dx$$

Problem 41: Unable to integrate problem.

$$\int (e + f x)^m (a + b \text{ArcTan}[c + d x]) \, dx$$

Optimal (type 5, 177 leaves, 6 steps):

$$\frac{(e+f x)^{1+m}\,(a+b\,\mathrm{ArcTan}[c+d x])}{f\,(1+m)}-\frac{i\,b\,d\,(e+f x)^{2+m}\,\mathrm{Hypergeometric2F1}\left[1,\,2+m,\,3+m,\,\frac{d\,(e+f x)}{d e+i f-c f}\right]}{2 f\,(d e+(i-c) f)\,(1+m)\,(2+m)}+\frac{i\,b\,d\,(e+f x)^{2+m}\,\mathrm{Hypergeometric2F1}\left[1,\,2+m,\,3+m,\,\frac{d\,(e+f x)}{d e-(i+c) f}\right]}{2 f\,(d e-(i+c) f)\,(1+m)\,(2+m)}$$

Result (type 8, 20 leaves) :

$$\int (e+f x)^m\,(a+b\,\mathrm{ArcTan}[c+d x])\,dx$$

Problem 52: Result is not expressed in closed-form.

$$\int \frac{\mathrm{ArcTan}[a+b x]}{c+d x^3}\,dx$$

Optimal (type 4, 863 leaves, 23 steps) :

$$-\frac{i\,\mathrm{Log}[1+i a+i b x]\,\mathrm{Log}\left[\frac{b\,(c^{1/3}+d^{1/3} x)}{b c^{1/3}+(i-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}+\frac{i\,\mathrm{Log}[1-i a-i b x]\,\mathrm{Log}\left[\frac{b\,(c^{1/3}+d^{1/3} x)}{b c^{1/3}-(i+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}+$$
$$\frac{(-1)^{1/6}\,\mathrm{Log}[1+i a+i b x]\,\mathrm{Log}\left[\frac{b\,(c^{1/3}-(-1)^{1/3} d^{1/3} x)}{b c^{1/3}-(-1)^{1/3} (i-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}-$$
$$\frac{(-1)^{1/6}\,\mathrm{Log}[1-i a-i b x]\,\mathrm{Log}\left[\frac{b\,(c^{1/3}-(-1)^{1/3} d^{1/3} x)}{b c^{1/3}+(-1)^{1/3} (i+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}+$$
$$\frac{(-1)^{5/6}\,\mathrm{Log}[1+i a+i b x]\,\mathrm{Log}\left[\frac{b\,(c^{1/3}+(-1)^{2/3} d^{1/3} x)}{b c^{1/3}+(-1)^{2/3} (i-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}-$$
$$\frac{(-1)^{5/6}\,\mathrm{Log}[1-i a-i b x]\,\mathrm{Log}\left[\frac{b\,(c^{1/3}+(-1)^{2/3} d^{1/3} x)}{b c^{1/3}+(-1)^{1/6} (1-i a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}-$$
$$\frac{i\,\mathrm{PolyLog}\left[2,\,\frac{d^{1/3}\,(i-a-b x)}{b c^{1/3}+(i-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}+\frac{(-1)^{5/6}\,\mathrm{PolyLog}\left[2,\,-\frac{(-1)^{1/6} d^{1/3}\,(i-a-b x)}{i b c^{1/3}-(-1)^{1/6} (i-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}+$$
$$\frac{(-1)^{1/6}\,\mathrm{PolyLog}\left[2,\,-\frac{(-1)^{1/3} d^{1/3}\,(i-a-b x)}{b c^{1/3}-(-1)^{1/3} (i-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}+\frac{i\,\mathrm{PolyLog}\left[2,\,-\frac{d^{1/3}\,(i+a+b x)}{b c^{1/3}-(i+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}-$$
$$\frac{(-1)^{1/6}\,\mathrm{PolyLog}\left[2,\,\frac{(-1)^{1/3} d^{1/3}\,(i+a+b x)}{b c^{1/3}+(-1)^{1/3} (i+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}-\frac{(-1)^{5/6}\,\mathrm{PolyLog}\left[2,\,-\frac{(-1)^{2/3} d^{1/3}\,(i+a+b x)}{b c^{1/3}-(-1)^{2/3} (i+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}}$$

Result (type 7, 892 leaves) :

$$-\frac{1}{6} b^2 \text{RootSum}\Big[b^3 c - i d + 3 a d + 3 i a^2 d - a^3 d + 3 b^3 c \#1 + 3 i d \#1 - 3 a d \#1 + 3 i a^2 d \#1 - 3 a^3 d \#1 + 3 b^3 c \#1^2 - 3 i d \#1^2 - 3 a d \#1^2 - 3 i a^2 d \#1^2 - 3 a^3 d \#1^2 + b^3 c \#1^3 + i d \#1^3 + 3 a d \#1^3 - 3 i a^2 d \#1^3 - a^3 d \#1^3 \&,$$

$$\Big(-\pi \text{ArcTan}[a+b x] - 2 \text{ArcTan}[a+b x]^2 + 2 i \text{ArcTan}[a+b x] \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right] + i \pi \text{Log}\left[1+e^{-2 i \text{ArcTan}[a+b x]}\right] + 2 i \text{ArcTan}[a+b x] \text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right] - 2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right] \text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right] - i \pi \text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right] + 2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right] \text{Log}\left[\text{Sin}\left[\text{ArcTan}[a+b x] + i \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\right]\right] + \text{PolyLog}\left[2, e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right] - 2 \text{ArcTan}[a+b x]^2 \#1 + \pi \text{ArcTan}[a+b x] \#1^2 - 2 i \text{ArcTan}[a+b x] \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right] \#1^2 - i \pi \text{Log}\left[1+e^{-2 i \text{ArcTan}[a+b x]}\right] \#1^2 - 2 i \text{ArcTan}[a+b x] \text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right] \#1^2 + 2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right] \text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right] \#1^2 + i \pi \text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right] \#1^2 - 2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right] \text{Log}\left[\text{Sin}\left[\text{ArcTan}[a+b x] + i \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\right]\right] \#1^2 - \text{PolyLog}\left[2, e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right] \#1^2 + 2 e^{\text{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]} \text{ArcTan}[a+b x]^2 \sqrt{\frac{\#1}{(1+\#1)^2}} + 4 e^{\text{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]} \text{ArcTan}[a+b x]^2 \#1 \sqrt{\frac{\#1}{(1+\#1)^2}} + 2 e^{\text{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]} \text{ArcTan}[a+b x]^2 \#1^2 \sqrt{\frac{\#1}{(1+\#1)^2}}\Big) \Big/ \left(b^3 c + a d + 2 i a^2 d - a^3 d + 2 b^3 c \#1 - 2 a d \#1 - 2 a^3 d \#1 + b^3 c \#1^2 + a d \#1^2 - 2 i a^2 d \#1^2 - a^3 d \#1^2\right) \&\Big]$$

Problem 53: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTan}[a+b x]}{c+d x^2} \, dx$$

Optimal (type 4, 543 leaves, 17 steps):

$$-\frac{i \,\text{Log}[1+i a+i b x] \,\text{Log}\left[\frac{b\left(\sqrt{-c}-\sqrt{d} x\right)}{b \sqrt{-c}-(i-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{i \,\text{Log}[1-i a-i b x] \,\text{Log}\left[\frac{b\left(\sqrt{-c}-\sqrt{d} x\right)}{b \sqrt{-c}+(i+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+$$

$$\frac{i \,\text{Log}[1+i a+i b x] \,\text{Log}\left[\frac{b\left(\sqrt{-c}+\sqrt{d} x\right)}{b \sqrt{-c}+(i-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}-\frac{i \,\text{Log}[1-i a-i b x] \,\text{Log}\left[\frac{b\left(\sqrt{-c}+\sqrt{d} x\right)}{b \sqrt{-c}-(i+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}-$$

$$\frac{i \,\text{PolyLog}\left[2,-\frac{\sqrt{d}(i-a-b x)}{b \sqrt{-c}-(i-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{i \,\text{PolyLog}\left[2, \frac{\sqrt{d}(i-a-b x)}{b \sqrt{-c}+(i-a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}-$$

$$\frac{i \,\text{PolyLog}\left[2,-\frac{\sqrt{d}(i+a+b x)}{b \sqrt{-c}-(i+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}+\frac{i \,\text{PolyLog}\left[2, \frac{\sqrt{d}(i+a+b x)}{b \sqrt{-c}+(i+a) \sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}$$

Result (type 4, 1501 leaves):

$$\frac{1}{4\left(1+a^2\right) \sqrt{c}\, d}$$

$$\left(-2 \sqrt{d} \,\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]-2 a^2 \sqrt{d} \,\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]+\right.$$

$$2 \sqrt{d} \,\text{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]+2 a^2 \sqrt{d} \,\text{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]-$$

$$2 b \sqrt{c} \,\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2+b \sqrt{c} \sqrt{\frac{b^2 c+(-i+a)^2 d}{b^2 c}}\, e^{-i \,\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2-$$

$$i a b \sqrt{c} \sqrt{\frac{b^2 c+(-i+a)^2 d}{b^2 c}}\, e^{-i \,\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2+$$

$$b \sqrt{c} \sqrt{\frac{b^2 c+(i+a)^2 d}{b^2 c}}\, e^{-i \,\text{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2+$$

$$i a b \sqrt{c} \sqrt{\frac{b^2 c+(i+a)^2 d}{b^2 c}}\, e^{-i \,\text{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]^2+$$

$$4\left(1+a^2\right) \sqrt{d} \,\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \text{ArcTan}[a+b x]+$$

$$2 i \sqrt{d} \,\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] \text{Log}\left[1-e^{-2 i\left(\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right]+$$

$$2 i a^2 \sqrt{d} \,\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] \text{Log}\left[1-e^{-2 i\left(\text{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right]+$$

$$2\,i\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\,\text{Log}\left[1-e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(-i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]+$$

$$2\,i\,a^2\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\,\text{Log}\left[1-e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(-i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-$$

$$2\,i\,\sqrt{d}\,\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]\,\text{Log}\left[1-e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-$$

$$2\,i\,a^2\,\sqrt{d}\,\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]\,\text{Log}\left[1-e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-$$

$$2\,i\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\,\text{Log}\left[1-e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-$$

$$2\,i\,a^2\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\,\text{Log}\left[1-e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-$$

$$2\,i\,\sqrt{d}\,\text{ArcTan}\left[\frac{(-i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(-i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]-$$

$$2\,i\,a^2\,\sqrt{d}\,\text{ArcTan}\left[\frac{(-i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(-i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]+$$

$$2\,i\,\sqrt{d}\,\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]+$$

$$2\,i\,a^2\,\sqrt{d}\,\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]-$$

$$\left(1+a^2\right)\,\sqrt{d}\,\text{PolyLog}\left[2,\,e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(-i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]+$$

$$\left.\left(1+a^2\right)\,\sqrt{d}\,\text{PolyLog}\left[2,\,e^{-2\,i\,\left(\text{ArcTan}\left[\frac{(i+a)\,\sqrt{d}}{b\,\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]\right)$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTan}[a+b\,x]}{c+d\,x}\,dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$-\frac{\text{ArcTan}[a+b\,x]\,\text{Log}\left[\frac{2}{1-i\,(a+b\,x)}\right]}{d}+\frac{\text{ArcTan}[a+b\,x]\,\text{Log}\left[\frac{2\,b\,(c+d\,x)}{(b\,c+i\,d-a\,d)\,(1-i\,(a+b\,x))}\right]}{d}+$$

$$\frac{i\,\text{PolyLog}\left[2,\,1-\frac{2}{1-i\,(a+b\,x)}\right]}{2\,d}-\frac{i\,\text{PolyLog}\left[2,\,1-\frac{2\,b\,(c+d\,x)}{(b\,c+i\,d-a\,d)\,(1-i\,(a+b\,x))}\right]}{2\,d}$$

Result (type 4, 305 leaves):

$$\frac{1}{d}\left(\text{ArcTan}[a+b x]\left(-\text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]+\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{b c-a d}{d}\right]+\text{ArcTan}[a+b x]\right]\right]\right)+\frac{1}{2}\left(-\frac{1}{4} i (\pi-2\,\text{ArcTan}[a+b x])^2-i\left(\text{ArcTan}\left[\frac{b c-a d}{d}\right]+\text{ArcTan}[a+b x]\right)^2+(\pi-2\,\text{ArcTan}[a+b x])\,\text{Log}\left[1+e^{-2 i\,\text{ArcTan}[a+b x]}\right]+2\left(\text{ArcTan}\left[\frac{b c-a d}{d}\right]+\text{ArcTan}[a+b x]\right)\text{Log}\left[1-e^{2 i\left(\text{ArcTan}\left[\frac{b c-a d}{d}\right]+\text{ArcTan}[a+b x]\right)}\right]-(\pi-2\,\text{ArcTan}[a+b x])\,\text{Log}\left[\frac{2}{\sqrt{1+(a+b x)^2}}\right]-2\left(\text{ArcTan}\left[\frac{b c-a d}{d}\right]+\text{ArcTan}[a+b x]\right)\text{Log}\left[2\,\text{Sin}\left[\text{ArcTan}\left[\frac{b c-a d}{d}\right]+\text{ArcTan}[a+b x]\right]\right]-i\,\text{PolyLog}\left[2,-e^{-2 i\,\text{ArcTan}[a+b x]}\right]-i\,\text{PolyLog}\left[2,e^{2 i\left(\text{ArcTan}\left[\frac{b c-a d}{d}\right]+\text{ArcTan}[a+b x]\right)}\right]\right)\right)$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTan}[a+b x]}{c+\frac{d}{x}}\,dx$$

Optimal (type 4, 244 leaves, 15 steps):

$$-\frac{(1+i a+i b x)\,\text{Log}[1+i a+i b x]}{2 b c}-\frac{(1-i a-i b x)\,\text{Log}[-i\,(i+a+b x)]}{2 b c}-\frac{i d\,\text{Log}[1-i a-i b x]\,\text{Log}\left[-\frac{b (d+c x)}{(i+a) c-b d}\right]}{2 c^2}+\frac{i d\,\text{Log}[1+i a+i b x]\,\text{Log}\left[\frac{b (d+c x)}{(i-a) c+b d}\right]}{2 c^2}+\frac{i d\,\text{PolyLog}\left[2,\frac{c (i-a-b x)}{i c-a c+b d}\right]}{2 c^2}-\frac{i d\,\text{PolyLog}\left[2,\frac{c (i+a+b x)}{(i+a) c-b d}\right]}{2 c^2}$$

Result (type 4, 771 leaves):

$$\frac{1}{b c^2 \left(-2 a c+2 b d\right)}\left(-2 a^2 c^2 \text{ArcTan}[a+b x]+2 a b c d \text{ArcTan}[a+b x]+\right.$$
$$i\, a b c d \pi \text{ArcTan}[a+b x]-i\, b^2 d^2 \pi \text{ArcTan}[a+b x]-2 a b c^2 x \text{ArcTan}[a+b x]+$$
$$2 b^2 c d x \text{ArcTan}[a+b x]+2 i\, a b c d \text{ArcTan}\left[a-\frac{b d}{c}\right] \text{ArcTan}[a+b x]-$$
$$2 i\, b^2 d^2 \text{ArcTan}\left[a-\frac{b d}{c}\right] \text{ArcTan}[a+b x]-b c d \text{ArcTan}[a+b x]^2+i\, a b c d \text{ArcTan}[a+b x]^2-$$
$$i\, b^2 d^2 \text{ArcTan}[a+b x]^2+b c d \sqrt{1+a^2-\frac{2 a b d}{c}+\frac{b^2 d^2}{c^2}}\; e^{-i \text{ArcTan}\left[a-\frac{b d}{c}\right]} \text{ArcTan}[a+b x]^2+$$
$$a b c d \pi \text{Log}\left[1+e^{-2 i \text{ArcTan}[a+b x]}\right]-b^2 d^2 \pi \text{Log}\left[1+e^{-2 i \text{ArcTan}[a+b x]}\right]-$$
$$2 a b c d \text{ArcTan}[a+b x] \text{Log}\left[1+e^{2 i \text{ArcTan}[a+b x]}\right]+2 b^2 d^2 \text{ArcTan}[a+b x] \text{Log}\left[1+e^{2 i \text{ArcTan}[a+b x]}\right]-$$
$$2 a b c d \text{ArcTan}\left[a-\frac{b d}{c}\right] \text{Log}\left[1-e^{2 i \left(-\text{ArcTan}\left[a-\frac{b d}{c}\right]+\text{ArcTan}[a+b x]\right)}\right]+$$
$$2 b^2 d^2 \text{ArcTan}\left[a-\frac{b d}{c}\right] \text{Log}\left[1-e^{2 i \left(-\text{ArcTan}\left[a-\frac{b d}{c}\right]+\text{ArcTan}[a+b x]\right)}\right]+$$
$$2 a b c d \text{ArcTan}[a+b x] \text{Log}\left[1-e^{2 i \left(-\text{ArcTan}\left[a-\frac{b d}{c}\right]+\text{ArcTan}[a+b x]\right)}\right]-$$
$$2 b^2 d^2 \text{ArcTan}[a+b x] \text{Log}\left[1-e^{2 i \left(-\text{ArcTan}\left[a-\frac{b d}{c}\right]+\text{ArcTan}[a+b x]\right)}\right]-2 a c^2 \text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]+$$
$$2 b c d \text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]-a b c d \pi \text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]+b^2 d^2 \pi \text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]+$$
$$2 a b c d \text{ArcTan}\left[a-\frac{b d}{c}\right] \text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[a-\frac{b d}{c}\right]-\text{ArcTan}[a+b x]\right]\right]-$$
$$2 b^2 d^2 \text{ArcTan}\left[a-\frac{b d}{c}\right] \text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[a-\frac{b d}{c}\right]-\text{ArcTan}[a+b x]\right]\right]+$$
$$i\, b d (a c-b d) \text{PolyLog}\left[2,-e^{2 i \text{ArcTan}[a+b x]}\right]+$$
$$\left.i\, b d (-a c+b d) \text{PolyLog}\left[2, e^{2 i \left(-\text{ArcTan}\left[a-\frac{b d}{c}\right]+\text{ArcTan}[a+b x]\right)}\right]\right)$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTan}[a+b x]}{c+\frac{d}{x^2}}\, dx$$

Optimal (type 4, 668 leaves, 25 steps):

$$-\frac{(1+i a+i b x)\,\text{Log}[1+i a+i b x]}{2 b c}-\frac{(1-i a-i b x)\,\text{Log}[-i\,(i+a+b x)]}{2 b c}+$$
$$\frac{i\sqrt{d}\,\text{Log}[1+i a+i b x]\,\text{Log}\Big[-\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}-$$
$$\frac{i\sqrt{d}\,\text{Log}[1-i a-i b x]\,\text{Log}\Big[\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}+$$
$$\frac{i\sqrt{d}\,\text{Log}[1-i a-i b x]\,\text{Log}\Big[-\frac{b\left(\sqrt{d}+\sqrt{-c}\,x\right)}{(i+a)\sqrt{-c}-b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}-\frac{i\sqrt{d}\,\text{Log}[1+i a+i b x]\,\text{Log}\Big[\frac{b\left(\sqrt{d}+\sqrt{-c}\,x\right)}{i\sqrt{-c}-a\sqrt{-c}+b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}+$$
$$\frac{i\sqrt{d}\,\text{PolyLog}\Big[2,\frac{\sqrt{-c}\,(i-a-b x)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}-\frac{i\sqrt{d}\,\text{PolyLog}\Big[2,\frac{\sqrt{-c}\,(1+i a+i b x)}{(1+i a)\sqrt{-c}-i b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}+$$
$$\frac{i\sqrt{d}\,\text{PolyLog}\Big[2,\frac{\sqrt{-c}\,(i+a+b x)}{i\sqrt{-c}+a\sqrt{-c}-b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}-\frac{i\sqrt{d}\,\text{PolyLog}\Big[2,\frac{\sqrt{-c}\,(i+a+b x)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\Big]}{4\,(-c)^{3/2}}$$

Result (type 4, 1536 leaves):

$$\frac{(a+b x)\,\text{ArcTan}[a+b x]+\text{Log}\Big[\frac{1}{\sqrt{1+(a+b x)^2}}\Big]}{b c}-\frac{1}{4\,(1+a^2)\,c^2}\sqrt{d}$$
$$\Bigg(-2\sqrt{c}\,\text{ArcTan}\Big[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]-2 a^2\sqrt{c}\,\text{ArcTan}\Big[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]+$$
$$2\sqrt{c}\,\text{ArcTan}\Big[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]+2 a^2\sqrt{c}\,\text{ArcTan}\Big[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]-$$
$$2 b\sqrt{d}\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]^2+b\sqrt{d}\,\sqrt{\frac{(-i+a)^2 c+b^2 d}{b^2 d}}\;e^{-i\,\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]^2-$$
$$i a b\sqrt{d}\,\sqrt{\frac{(-i+a)^2 c+b^2 d}{b^2 d}}\;e^{-i\,\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]^2+$$
$$b\sqrt{d}\,\sqrt{\frac{(i+a)^2 c+b^2 d}{b^2 d}}\;e^{-i\,\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]^2+i a b\sqrt{d}\,\sqrt{\frac{(i+a)^2 c+b^2 d}{b^2 d}}$$
$$e^{-i\,\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]^2+4\,(1+a^2)\sqrt{c}\,\text{ArcTan}\Big[\frac{\sqrt{c}\,x}{\sqrt{d}}\Big]\,\text{ArcTan}[a+b x]+$$
$$2 i\sqrt{c}\,\text{ArcTan}\Big[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\Big]\,\text{Log}\Big[1-e^{-2 i\left(\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\Big]+$$

$$2\,i\,a^2\sqrt{c}\,\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]+$$

$$2\,i\sqrt{c}\,\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]+$$

$$2\,i\,a^2\sqrt{c}\,\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]-$$

$$2\,i\sqrt{c}\,\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]-$$

$$2\,i\,a^2\sqrt{c}\,\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]-$$

$$2\,i\sqrt{c}\,\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]-$$

$$2\,i\,a^2\sqrt{c}\,\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]-$$

$$2\,i\sqrt{c}\,\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right]\right]-$$

$$2\,i\,a^2\sqrt{c}\,\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right]\right]+$$

$$2\,i\sqrt{c}\,\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right]\right]+$$

$$2\,i\,a^2\sqrt{c}\,\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right]\right]-$$

$$\left(1+a^2\right)\sqrt{c}\,\text{PolyLog}\left[2,\,e^{-2\,i\left(\text{ArcTan}\left[\frac{(-i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]+$$

$$\left.\left(1+a^2\right)\sqrt{c}\,\text{PolyLog}\left[2,\,e^{-2\,i\left(\text{ArcTan}\left[\frac{(i+a)\sqrt{c}}{b\sqrt{d}}\right]+\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\right)}\right]\right)$$

Problem 57: Result is not expressed in closed-form.

$$\int \frac{\text{ArcTan}[a+b\,x]}{c+\frac{d}{x^3}}\,dx$$

Optimal (type 4, 933 leaves, 31 steps):

$$-\frac{(1+i a+i b x)\,\text{Log}[1+i a+i b x]}{2 b c}-\frac{(1-i a-i b x)\,\text{Log}[-i\,(i+a+b x)]}{2 b c}-\frac{i\,d^{1/3}\,\text{Log}[1-i a-i b x]\,\text{Log}\left[-\frac{b\left(d^{1/3}+c^{1/3} x\right)}{(i+a)\,c^{1/3}-b\,d^{1/3}}\right]}{6 c^{4/3}}+\frac{i\,d^{1/3}\,\text{Log}[1+i a+i b x]\,\text{Log}\left[\frac{b\left(d^{1/3}+c^{1/3} x\right)}{(i-a)\,c^{1/3}+b\,d^{1/3}}\right]}{6 c^{4/3}}-\frac{(-1)^{1/6}\,d^{1/3}\,\text{Log}[1+i a+i b x]\,\text{Log}\left[-\frac{b\left(d^{1/3}-(-1)^{1/3} c^{1/3} x\right)}{(-1)^{1/3}\,(i-a)\,c^{1/3}-b\,d^{1/3}}\right]}{6 c^{4/3}}+\frac{(-1)^{1/6}\,d^{1/3}\,\text{Log}[1-i a-i b x]\,\text{Log}\left[\frac{b\left(d^{1/3}-(-1)^{1/3} c^{1/3} x\right)}{(-1)^{1/3}\,(i+a)\,c^{1/3}+b\,d^{1/3}}\right]}{6 c^{4/3}}-\frac{(-1)^{5/6}\,d^{1/3}\,\text{Log}[1+i a+i b x]\,\text{Log}\left[\frac{b\left(d^{1/3}+(-1)^{2/3} c^{1/3} x\right)}{(-1)^{2/3}\,(i-a)\,c^{1/3}+b\,d^{1/3}}\right]}{6 c^{4/3}}+\frac{(-1)^{5/6}\,d^{1/3}\,\text{Log}[1-i a-i b x]\,\text{Log}\left[\frac{b\left(d^{1/3}+(-1)^{2/3} c^{1/3} x\right)}{(-1)^{1/6}\,(1-i a)\,c^{1/3}+b\,d^{1/3}}\right]}{6 c^{4/3}}-\frac{(-1)^{1/6}\,d^{1/3}\,\text{PolyLog}\left[2,\frac{(-1)^{1/3} c^{1/3}\,(i-a-b x)}{(-1)^{1/3}\,(i-a)\,c^{1/3}-b\,d^{1/3}}\right]}{6 c^{4/3}}-\frac{(-1)^{5/6}\,d^{1/3}\,\text{PolyLog}\left[2,\frac{(-1)^{1/6} c^{1/3}\,(i-a-b x)}{(-1)^{1/6}\,(i-a)\,c^{1/3}-i\,b\,d^{1/3}}\right]}{6 c^{4/3}}+\frac{i\,d^{1/3}\,\text{PolyLog}\left[2,\frac{c^{1/3}\,(i-a-b x)}{(i-a)\,c^{1/3}+b\,d^{1/3}}\right]}{6 c^{4/3}}-\frac{i\,d^{1/3}\,\text{PolyLog}\left[2,\frac{c^{1/3}\,(i+a+b x)}{(i+a)\,c^{1/3}-b\,d^{1/3}}\right]}{6 c^{4/3}}+\frac{(-1)^{5/6}\,d^{1/3}\,\text{PolyLog}\left[2,\frac{(-1)^{2/3} c^{1/3}\,(i+a+b x)}{(-1)^{2/3}\,(i+a)\,c^{1/3}-b\,d^{1/3}}\right]}{6 c^{4/3}}+\frac{(-1)^{1/6}\,d^{1/3}\,\text{PolyLog}\left[2,\frac{(-1)^{1/3} c^{1/3}\,(i+a+b x)}{(-1)^{1/3}\,(i+a)\,c^{1/3}+b\,d^{1/3}}\right]}{6 c^{4/3}}$$

Result (type 7, 933 leaves) :

$$\frac{1}{6 b c}\left(6\left((a+b x)\,\text{ArcTan}[a+b x]+\text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]\right)-\right.$$
$$b^3 d\,\text{RootSum}\Big[i c-3 a c-3 i a^2 c+a^3 c-b^3 d-3 i c \#1+3 a c \#1-3 i a^2 c \#1+3 a^3 c \#1-3 b^3 d \#1+3 i c \#1^2+3 a c \#1^2+3 i a^2 c \#1^2+3 a^3 c \#1^2-3 b^3 d \#1^2-i c \#1^3-3 a c \#1^3+3 i a^2 c \#1^3+a^3 c \#1^3-b^3 d \#1^3\,\&,$$
$$\Big(-\pi\,\text{ArcTan}[a+b x]-2\,\text{ArcTan}[a+b x]^2+2 i\,\text{ArcTan}[a+b x]\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]+$$
$$i \pi\,\text{Log}\left[1+e^{-2 i \text{ArcTan}[a+b x]}\right]+2 i\,\text{ArcTan}[a+b x]\,\text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right]-$$
$$2\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right]-i \pi\,\text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]+$$
$$2\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\text{Log}\left[\text{Sin}\left[\text{ArcTan}[a+b x]+i\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\right]\right]+$$
$$\text{PolyLog}\left[2,e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right]-2\,\text{ArcTan}[a+b x]^2\,\#1+\pi\,\text{ArcTan}[a+b x]\,\#1^2-$$
$$2 i\,\text{ArcTan}[a+b x]\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\#1^2-i \pi\,\text{Log}\left[1+e^{-2 i \text{ArcTan}[a+b x]}\right]\#1^2-$$
$$2 i\,\text{ArcTan}[a+b x]\,\text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right]\#1^2+2\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]$$
$$\text{Log}\left[1-e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right]\#1^2+i \pi\,\text{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right]\#1^2-$$
$$2\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\text{Log}\left[\text{Sin}\left[\text{ArcTan}[a+b x]+i\,\text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]\right]\right]\#1^2-$$
$$\text{PolyLog}\left[2,e^{2 i \text{ArcTan}[a+b x]-2 \text{ArcTanh}\left[\frac{-1+\#1}{1+\#1}\right]}\right]\#1^2+2 e^{\text{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]}\text{ArcTan}[a+b x]^2$$
$$\sqrt{\frac{\#1}{(1+\#1)^2}}+4 e^{\text{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]}\text{ArcTan}[a+b x]^2\,\#1\sqrt{\frac{\#1}{(1+\#1)^2}}+$$
$$2 e^{\text{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]}\text{ArcTan}[a+b x]^2\,\#1^2\sqrt{\frac{\#1}{(1+\#1)^2}}\Big)\Big/\left(-a c-2 i a^2 c+a^3 c-b^3 d+\right.$$
$$\left.\left.2 a c \#1+2 a^3 c \#1-2 b^3 d \#1-a c \#1^2+2 i a^2 c \#1^2+a^3 c \#1^2-b^3 d \#1^2\right)\,\&\Big]\right)$$

Problem 58: Result is not expressed in closed-form.

$$\int \frac{\text{ArcTan}[a+b x]}{c+d\sqrt{x}}\,dx$$

Optimal (type 4, 673 leaves, 31 steps):

$$\frac{2\,i\sqrt{i+a}\,\text{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{b}\,d}-\frac{2\,i\sqrt{i-a}\,\text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right]}{\sqrt{b}\,d}+$$

$$\frac{i\,c\,\text{Log}\left[\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}-\frac{i\,c\,\text{Log}\left[\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}+$$

$$\frac{i\,c\,\text{Log}\left[-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-i-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}-\frac{i\,c\,\text{Log}\left[-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{i-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}+$$

$$\frac{i\sqrt{x}\,\text{Log}[1-i\,a-i\,b\,x]}{d}-\frac{i\,c\,\text{Log}\left[c+d\sqrt{x}\right]\text{Log}[1-i\,a-i\,b\,x]}{d^2}-\frac{i\sqrt{x}\,\text{Log}[1+i\,a+i\,b\,x]}{d}+$$

$$\frac{i\,c\,\text{Log}\left[c+d\sqrt{x}\right]\text{Log}[1+i\,a+i\,b\,x]}{d^2}+\frac{i\,c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-i-a}\,d}\right]}{d^2}+$$

$$\frac{i\,c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right]}{d^2}-\frac{i\,c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{i-a}\,d}\right]}{d^2}-\frac{i\,c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right]}{d^2}$$

Result (type 7, 303 leaves):

$$\frac{1}{2\,d^2}\left(-\frac{4\,i\sqrt{-i+a}\,d\,\text{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{-i+a}}\right]}{\sqrt{b}}+\frac{4\,i\sqrt{i+a}\,d\,\text{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{b}}+4\,\text{ArcTan}[a+b\,x]\left(d\sqrt{x}-c\,\text{Log}\left[c+d\sqrt{x}\right]\right)+\right.$$
$$c\,d^2\,\text{RootSum}\left[b^2\,c^4+2\,a\,b\,c^2\,d^2+d^4+a^2\,d^4-4\,b^2\,c^3\,\#1-4\,a\,b\,c\,d^2\,\#1+6\,b^2\,c^2\,\#1^2+2\,a\,b\,d^2\,\#1^2-4\,b^2\,c\,\#1^3+b^2\,\#1^4\,\&,\right.$$
$$\left.\left.\left(-\text{Log}\left[c+d\sqrt{x}\right]^2+2\,\text{Log}\left[c+d\sqrt{x}\right]\text{Log}\left[1-\frac{c+d\sqrt{x}}{\#1}\right]+2\,\text{PolyLog}\left[2,\frac{c+d\sqrt{x}}{\#1}\right]\right)\Big/\left(b\,c^2+a\,d^2-2\,b\,c\,\#1+b\,\#1^2\right)\,\&\right]\right)$$

Problem 59: Unable to integrate problem.

$$\int\frac{\text{ArcTan}[a+b\,x]}{c+\frac{d}{\sqrt{x}}}\,dx$$

Optimal (type 4, 770 leaves, 37 steps):

$$-\frac{2\,i\,\sqrt{i+a}\,d\,\text{ArcTan}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{b}\,c^2}+\frac{2\,i\,\sqrt{i-a}\,d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right]}{\sqrt{b}\,c^2}-$$

$$\frac{i\,d^2\,\text{Log}\left[\frac{c\left(\sqrt{-i-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right]\text{Log}\left[d+c\,\sqrt{x}\right]}{c^3}+\frac{i\,d^2\,\text{Log}\left[\frac{c\left(\sqrt{i-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{i-a}\,c+\sqrt{b}\,d}\right]\text{Log}\left[d+c\,\sqrt{x}\right]}{c^3}-$$

$$\frac{i\,d^2\,\text{Log}\left[\frac{c\left(\sqrt{-i-a}+\sqrt{b}\,\sqrt{x}\right)}{\sqrt{-i-a}\,c-\sqrt{b}\,d}\right]\text{Log}\left[d+c\,\sqrt{x}\right]}{c^3}+\frac{i\,d^2\,\text{Log}\left[\frac{c\left(\sqrt{i-a}+\sqrt{b}\,\sqrt{x}\right)}{\sqrt{i-a}\,c-\sqrt{b}\,d}\right]\text{Log}\left[d+c\,\sqrt{x}\right]}{c^3}-$$

$$\frac{i\,d\,\sqrt{x}\,\text{Log}[1-i\,a-i\,b\,x]}{c^2}+\frac{i\,d^2\,\text{Log}\left[d+c\,\sqrt{x}\right]\text{Log}[1-i\,a-i\,b\,x]}{c^3}+\frac{i\,d\,\sqrt{x}\,\text{Log}[1+i\,a+i\,b\,x]}{c^2}-$$

$$\frac{(1+i\,a+i\,b\,x)\,\text{Log}[1+i\,a+i\,b\,x]}{2\,b\,c}-\frac{i\,d^2\,\text{Log}\left[d+c\,\sqrt{x}\right]\text{Log}[1+i\,a+i\,b\,x]}{c^3}-$$

$$\frac{(1-i\,a-i\,b\,x)\,\text{Log}[-i\,(i+a+b\,x)]}{2\,b\,c}-\frac{i\,d^2\,\text{PolyLog}\left[2,-\frac{\sqrt{b}\left(d+c\,\sqrt{x}\right)}{\sqrt{-i-a}\,c-\sqrt{b}\,d}\right]}{c^3}+$$

$$\frac{i\,d^2\,\text{PolyLog}\left[2,-\frac{\sqrt{b}\left(d+c\,\sqrt{x}\right)}{\sqrt{i-a}\,c-\sqrt{b}\,d}\right]}{c^3}-\frac{i\,d^2\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(d+c\,\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right]}{c^3}+\frac{i\,d^2\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(d+c\,\sqrt{x}\right)}{\sqrt{i-a}\,c+\sqrt{b}\,d}\right]}{c^3}$$

Result (type 8, 20 leaves) :

$$\int\frac{\text{ArcTan}[a+b\,x]}{c+\frac{d}{\sqrt{x}}}\,dx$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcTan}[d+e\,x]}{a+b\,x^2}\,dx$$

Optimal (type 4, 543 leaves, 17 steps) :

$$\frac{i\,\text{Log}\left[\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right]\text{Log}[1-i\,d-i\,e\,x]}{4\,\sqrt{-a}\,\sqrt{b}}-\frac{i\,\text{Log}\left[-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)-\sqrt{-a}\,e}\right]\text{Log}[1-i\,d-i\,e\,x]}{4\,\sqrt{-a}\,\sqrt{b}}-$$

$$\frac{i\,\text{Log}\left[-\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(i-d)-\sqrt{-a}\,e}\right]\text{Log}[1+i\,d+i\,e\,x]}{4\,\sqrt{-a}\,\sqrt{b}}+\frac{i\,\text{Log}\left[\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,(i-d)+\sqrt{-a}\,e}\right]\text{Log}[1+i\,d+i\,e\,x]}{4\,\sqrt{-a}\,\sqrt{b}}-$$

$$\frac{i\,\text{PolyLog}\left[2,\frac{\sqrt{b}\,(i-d-e\,x)}{\sqrt{b}\,(i-d)-\sqrt{-a}\,e}\right]}{4\,\sqrt{-a}\,\sqrt{b}}+\frac{i\,\text{PolyLog}\left[2,\frac{\sqrt{b}\,(i-d-e\,x)}{\sqrt{b}\,(i-d)+\sqrt{-a}\,e}\right]}{4\,\sqrt{-a}\,\sqrt{b}}-$$

$$\frac{i\,\text{PolyLog}\left[2,\frac{\sqrt{b}\,(i+d+e\,x)}{\sqrt{b}\,(i+d)-\sqrt{-a}\,e}\right]}{4\,\sqrt{-a}\,\sqrt{b}}+\frac{i\,\text{PolyLog}\left[2,\frac{\sqrt{b}\,(i+d+e\,x)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right]}{4\,\sqrt{-a}\,\sqrt{b}}$$

Result (type 4, 1501 leaves) :

$$\frac{1}{4\sqrt{a}\,b\,(1+d^2)}\Bigg(-2\sqrt{b}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]-2\sqrt{b}\,d^2\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]+$$
$$2\sqrt{b}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]+2\sqrt{b}\,d^2\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\Big]\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]-$$
$$2\sqrt{a}\,e\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]^2+\sqrt{a}\,e\sqrt{\frac{b\,(-i+d)^2+a\,e^2}{a\,e^2}}\;e^{-i\,\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]^2-$$
$$i\sqrt{a}\,d\,e\sqrt{\frac{b\,(-i+d)^2+a\,e^2}{a\,e^2}}\;e^{-i\,\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]^2+$$
$$\sqrt{a}\,e\sqrt{\frac{b\,(i+d)^2+a\,e^2}{a\,e^2}}\;e^{-i\,\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]^2+$$
$$i\sqrt{a}\,d\,e\sqrt{\frac{b\,(i+d)^2+a\,e^2}{a\,e^2}}\;e^{-i\,\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]^2+$$
$$4\sqrt{b}\,(1+d^2)\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]\,\text{ArcTan}[d+e\,x]+$$
$$2i\sqrt{b}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\Big]\,\text{Log}\Big[1-e^{-2i\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\Big]+$$
$$2i\sqrt{b}\,d^2\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\Big]\,\text{Log}\Big[1-e^{-2i\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\Big]+$$
$$2i\sqrt{b}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]\,\text{Log}\Big[1-e^{-2i\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\Big]+$$
$$2i\sqrt{b}\,d^2\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]\,\text{Log}\Big[1-e^{-2i\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\Big]-$$
$$2i\sqrt{b}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\Big]\,\text{Log}\Big[1-e^{-2i\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\Big]-$$
$$2i\sqrt{b}\,d^2\,\text{ArcTan}\Big[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\Big]\,\text{Log}\Big[1-e^{-2i\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\Big]-$$
$$2i\sqrt{b}\,\text{ArcTan}\Big[\frac{\sqrt{b}\,x}{\sqrt{a}}\Big]\,\text{Log}\Big[1-e^{-2i\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\Big]-$$

$$2\,i\,\sqrt{b}\,d^2\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\,\text{Log}\left[1-e^{-2\,i\,\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\right]-$$

$$2\,i\,\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right]\right]-$$

$$2\,i\,\sqrt{b}\,d^2\,\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right]\right]+$$

$$2\,i\,\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right]\right]+$$

$$2\,i\,\sqrt{b}\,d^2\,\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]\,\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right]\right]-$$

$$\sqrt{b}\,\left(1+d^2\right)\,\text{PolyLog}\left[2,\,e^{-2\,i\,\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(-i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\right]+$$

$$\left.\sqrt{b}\,\left(1+d^2\right)\,\text{PolyLog}\left[2,\,e^{-2\,i\,\left(\text{ArcTan}\left[\frac{\sqrt{b}\,(i+d)}{\sqrt{a}\,e}\right]+\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]\right)}\right]\right)$$

Problem 62: Attempted integration timed out after 120 seconds.

$$\int\frac{\text{ArcTan}[d+e\,x]}{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 367 leaves, 12 steps):

$$\frac{\text{ArcTan}[d+e\,x]\,\text{Log}\left[\frac{2\,e\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(2\,c\,(i-d)+\left(b-\sqrt{b^2-4\,a\,c}\right)\,e\right)\,(1-i\,(d+e\,x))}\right]}{\sqrt{b^2-4\,a\,c}}-$$

$$\frac{\text{ArcTan}[d+e\,x]\,\text{Log}\left[\frac{2\,e\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(2\,c\,(i-d)+\left(b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,(1-i\,(d+e\,x))}\right]}{\sqrt{b^2-4\,a\,c}}-$$

$$\frac{i\,\text{PolyLog}\left[2,\,1+\frac{2\,\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e-2\,c\,(d+e\,x)\right)}{\left(2\,i\,c-2\,c\,d+b\,e-\sqrt{b^2-4\,a\,c}\,e\right)\,(1-i\,(d+e\,x))}\right]}{2\,\sqrt{b^2-4\,a\,c}}+$$

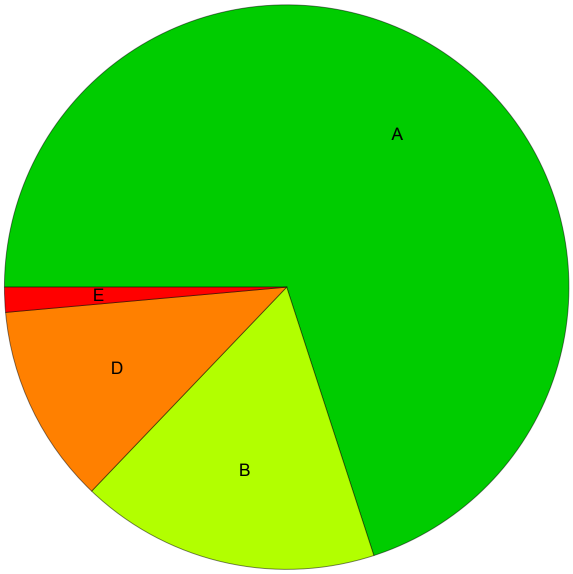
$$\frac{i\,\text{PolyLog}\left[2,\,1+\frac{2\,\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e-2\,c\,(d+e\,x)\right)}{\left(2\,c\,(i-d)+\left(b+\sqrt{b^2-4\,a\,c}\right)\,e\right)\,(1-i\,(d+e\,x))}\right]}{2\,\sqrt{b^2-4\,a\,c}}$$

Result (type 1, 1 leaves):

???

Summary of Integration Test Results

70 integration problems



A - 49 optimal antiderivatives

B - 12 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 8 unable to integrate problems

E - 1 integration timeouts